



复旦大学数学科学学院 数学综合报告会

报告题目: Asymptotic Formulas for Negative Sobolev Norms

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地点: 光华楼东主楼2001

报告摘要:

This talk concerns the asymptotic behavior of negative Sobolev norms as the differentiability order tends to zero. For $p \in (1, \infty)$ and $\alpha \in (0, 1)$, the negative Sobolev space $W^{-\alpha, p}(\mathbb{R}^n)$ is realized through heat-kernel convolutions. Under a mild boundedness assumption, we establish that for functions belonging to the union of $W^{-\alpha, p}(\mathbb{R}^n)$, $\lim_{\alpha \rightarrow 0^+} \alpha \|f\|_{W^{-\alpha, p}}^p = \frac{2}{p} \|f\|_{L^p}^p$. The proof uses heat-kernel regularization, monotonicity, weak compactness in reflexive L^p spaces, and an Abelian--Tauberian argument. The result is further extended to a general measure space equipped with a family of sub-additive, bounded, and continuous operators. These results complement the classical Bourgain--Brezis--Mironescu and Maz'ya--Shaposhnikova limiting formulas for positive fractional Sobolev norms.

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