2022复旦-科大谱几何会议

Workshop on Spectral Geometry

会议手册

2022 年 12 月 3 日-4 日

目　录

一、会议简介

会议时间：2022 年 12 月 3 日——2022 年 12 月 4 日

上午 9:00-11:35，下午 1:30-4:55

会议地点：腾讯会议在线

会议 ID：647 9456 0026

学术委员会：

洪家兴　　傅吉祥　　李嘉禹　　麻希南

组委会：

王作勤　　中国科学技术大学

华波波　　复旦大学

主办单位：中国科学技术大学，复旦大学

报告人（按姓氏首字母排列）：

陈虎元　　江西师范大学

陈化　　　武汉大学

黄耿耿　　复旦大学

林勇　　　清华大学

刘博　　　华东师范大学

刘世平　　中国科学技术大学

楼元　　　上海交通大学

苗长兴　　北京应用物理与计算数学研究所

王鹏　　　福建师范大学

王学锋　　香港中文大学（深圳）

吴云辉　　清华大学

夏波　　　中国科学技术大学

熊金钢　　北京师范大学

张振雷　　首都师范大学

二、会议时间安排表

	12 月 3 日（周六）	主持人
上午 9:00-9:45	陈化	李嘉禹
10:00-10:45	刘博	陆志勤
10:50-11:35	吴云辉	
下午 1:30-2:15	楼元	麻希南
2:20-3:05	熊金钢	
3:20-4:05	林勇	张会春
4:10-4:55	夏波	

	12 月 4 日（周日）	主持人
上午 9:00-9:45	王学锋	傅吉祥
10:00-10:45	刘世平	周家足
10:50-11:35	陈虎元	
下午 1:30-2:15	张振雷	陈群
2:20-3:05	王鹏	
3:20-4:05	黄耿耿	洪家兴
4:10-4:55	苗长兴	

三、报告信息

报告题目：Asymptotic behaviour of Dirichlet eigenvalues for homogeneous Hormander operators and algebraic geometry approach

报告人：陈化

报告人所在单位：武汉大学

报告摘要：In this talk, we study the Dirichlet eigenvalue problem of homogeneous H\"{o}rmander operators $\triangle_{X}=\sum_{j=1}^{m}X_{j}^{2}$ on a bounded open domain containing the origin, where smooth vector fields $X_{1}, X_{2},\ldots, X_{m}$ (as linear differential operators) are linearly independent and satisfying H\"{o}rmander's condition and a suitable homogeneity property with respect to a family of non-isotropic dilations. Suppose that Ω is a smooth open bounded domain in $\mathbb{R}^n$ containing the origin. Combining some new subelliptic heat kernel estimates, and a famous resolution of singularities in algebraic geometry and some refined analysis involving convex geometry, we establish the explicit asymptotic behaviour $\lambda_k\approx k^{\frac{2}{Q_{0}}}(\ln k)^{-\frac{2d_{0}}{Q_{0}}}$ as $k\to +\infty$, where λ_k denotes the k-th Dirichlet eigenvalue of $\triangle_{X}$ on Ω, Q_{0} is a positive rational number, and d_{0} is a non-negative integer. Furthermore, we also give the optimal bounds of index Q_{0}, which depends on the homogeneous dimension associated with vector fields $X_{1}, X_{2},\ldots, X_{m}$.

报告题目：Family APS index theorem, equivariant higher spectral flow and eta form

报告人：刘博

报告人所在单位：华东师范大学

报告摘要：Atiyah-Patodi-Singer established the index theorem for manifold with boundary. The general family extension of the APS index theorem was given by Melrose and Piazza by using the b-calculus and the spectral section. The boundary term there is the eta form with perturbation. In this talk, we will generalize the spectral section and the eta form to the equivariant case for a fiberwise compact Lie group action and discuss the properties of them. We will also discuss some problems about the family index theorem with boundary.

报告题目：Degenerating hyperbolic surfaces and spectral gaps for large genus

报告人：吴云辉

报告人所在单位：清华大学

报告摘要：We study the differences of two consecutive eigenvalues $\lambda_{i}-\lambda_{i-1}$ up to $i=2g-2$ for the Laplacian on hyperbolic surfaces of genus g, and show that the supremum of such spectral gaps over the moduli space has infimum limit at least $\frac{1}{4}$ as genus goes to infinity. A min-max principle for eigenvalues on degenerating hyperbolic surfaces is also established. This is a joint work with Haohao Zhang and Xuwen Zhu.

报告题目：时间周期抛物算子主特征值简介

报告人：楼元

报告人所在单位：上海交通大学

报告摘要：该报告将讨论时间周期的二阶抛物算子主特征值的部分历史，近期进展以及在生物中的应用，内容包括小扩散系数的主特征值渐近行为、主特征值关于周期的单调性、主特征值的水平集拓扑结构，以及对基本再生数的应用等。

报告题目：Harmonic maps with finite hyperbolic distances to the Extreme Kerr

报告人：熊金钢

报告人所在单位：北京师范大学

报告摘要：We study harmonic maps with finite hyperbolic distances to the Extreme Kerr from domains in the 3d Euclidean space to the hyperbolic plane. We prove that such maps have unique tangent maps at the black hole horizon. This particularly completes the regularity problem of harmonic maps arising from stationary axi-symmetric solutions of the Einstein vacuum field equations with mutiple black holes, dating back to Weinstein 1989 and Li-Tian 1992. This is joint with Q. Han, M. Khuri and G. Weinstein.

报告题目：Analytic and Reidemeister torsions of digraphs

报告人：林勇

报告人所在单位：清华大学

报告摘要：We define the notions of Reidemeister torsion and analytic torsion for directed graphs by means of the path homology theory introduced by Grigoryan-Lin-Muranov-Yau. We prove the identity of the two notions of torsions as well as obtain formulas for torsions of Cartesian products and joins of digraphs. This is a joint work with Grigoryan and Yau.

报告题目：The spectrum of the linearized operator of the ground state of a system of Klein-Gordon equations

报告人：夏波

报告人所在单位：中国科学技术大学

报告摘要：In this talk, we will first review the existence of ground states for a system of Klein-Gordon equations. In order to study the stability of such solutions, we shall study the spectrum of the corresponding linearized operator. We will give a complete description of the spectrum set for this linearized operator (in the radial case): the existence of a unique negative eigenvalue, no resonance, no embedded eigenvalue and the spectral gap property. This is based on a joint work with Yan Cui and Kai Yang.

报告题目：Principal Spectral Theory and Variational Characterizations for Cooperative Systems with Nonlocal Diffusion

报告人：王学锋

报告人所在单位：香港中文大学（深圳）

报告摘要：We study a general class of cooperative systems with nonlocal diffusion operators that may or may not be coupled. These systems are either "strong" in cooperation or "strong" in the coupling of the nonlocal diffusion operators, and in the former case, diffusion may not occur in some of the components of the system at all. We prove results concerning the existence, uniqueness, multiplicity, variational characterizations of the principal eigenvalue of the system, the spectral bound, the essential spectrum, and the relationship between the sign of principal eigenvalue and the validity of the maximum principle. We do so using an elementary method, without resorting to Krein-Rutmen theorem. This is a joint work with Yuanhang Su and Ting Zhang.

报告题目：Nodal domain theorems on signed graphs

报告人：刘世平

报告人所在单位：中国科学技术大学

报告摘要：A signed graph is a graph whose edges are labelled by a signature. It serves as a simple model of discrete vector bundle. The fundamental ideas of balance and switching of signed graphs often leads to more systematic understanding of various parts of graph theory. In this talk, we will make use of this structure to establish nodal domain theorems for arbitrary symmetric matrices. This is an extension of the discrete nodal domain theorem of Davies, Gladwell, Leydold, and Stadler for symmetric matrices with non-positive off-diagonal entries (which they call generalized Laplacians). We also discuss further applications to the spectral theory of graph p-Laplacian. This is based on joint works with Chuanyuan Ge and Dong Zhang.

报告题目：Bounds of the sum of eigenvalues for Dirichlet problems involving nonlocal operators

报告人：陈虎元

报告人所在单位：江西师范大学

报告摘要：

In this talk, we first consider the eigenvalues $\{\lambda_i(\mu)\}_i$ of the Dirichlet problem

$$(-\Delta)^{s_1}u = \lambda((-\Delta)^{s_2}u + \mu u) \ \text{in} \ \Omega, \qquad u = 0 \ \text{in} \ \mathbb{R}^N \setminus \Omega,$$

where $0 < s_2 < s_1 < 1$, $N > 2s_1$ and $(-\Delta)^s$ is the fractional Laplacian operator defined in the Cauchy principle value sense.

Secondly, we show bounds for the sequence of eigenvalues $\{\lambda_i(\Omega)\}_i$ of the Dirichlet problem

$$L_\Delta u = \lambda u \ \text{in} \ \Omega, \qquad u = 0 \ \text{in} \ \mathbb{R}^N \setminus \Omega,$$

where L_Δ is the logarithmic Laplacian operator with Fourier transform symbol $2\ln|\zeta|$. The logarithmic Laplacian operator is not positively defined if the volume of the domain is large enough.

In this talk, we obtain the upper and lower bounds for the sum of the first k eigenvalues for above two type nonlocal operators by extending the Li-Yau method and Kröger's method respectively.

报告题目：An almost Euclidean isoperimetric inequality

报告人：张振雷

报告人所在单位：首都师范大学

报告摘要：Brendle proved a sharp isoperimetric inequality on noncompact manifolds with nonnegative Ricci curvature. We localize his argument to show a local almost Euclidean isoperimetric inequality under the assumption of almost nonnegative Ricci curvature and almost Euclidean volume.

报告题目：On minimal submanifolds in spheres immersed by first eigenfunctions

报告人：王鹏

报告人所在单位：福建师范大学

报告摘要：Minimal submanifolds in spheres immersed by first eigenfunctions (λ_1-immersed) appear in many places naturally. In this talk we will review some spectral properties of minimal submanifolds in spheres and applications in various geometric variational problems. Moreover, some examples and characterizations of λ_1-immersed minimal homogeneous 3-dim and 4-dim tori in spheres will be presented.

报告题目：Uniqueness of the non-trivial solutions of some degenerate Monge-Ampere equation

报告人：黄耿耿

报告人所在单位：复旦大学

报告摘要：

Consider the following degenerate elliptic Monge-Ampère equation

$$\begin{cases} \det D^2u = f(-u), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega. \end{cases} \qquad (1)$$

Under suitable structure conditions on $f(t)$, we can show u and the solutions of linearized equation has the same symmetry property as the domain. Then for special $f(t) = t^p$, $p > n$, uniqueness of the non-trivial solutions is given. This is a joint work with Cheng Tingzhi.

报告题目：函数谱几何在调和分析与 PDEs 中作用

报告人：苗长兴

报告人所在单位：北京应用物理与计算数学研究所

报告摘要：总所周知,自由色散方程解的 Fourier 变换支撑在 Gauss 曲率非零的光滑超曲面上，超曲面的几何曲率如何影响解在物理空间所发生的结构性干涉?这就导致了研究非线性色散方程、非线性波动方程的 Fourier 限制模方法。限制性定理的对偶形式-Strichartz 估计、解的频率支撑光滑超曲面上所导致了波包分解及相应的平方函数估计、decoupling 估计等为研究非线性色散方程提供了研究框架与方法.而对椭圆方程、抛物方程而言,解的 Fourier 变换谱不再落在几何曲率非平凡的超曲面上，转而视为属于超平面上具有谱几乎紧化的特征，此时解在物理空间具有平均特征或满足 Harnack 不等式(实质上提供了局部反向 Sobolev 不等式)。 这为研究椭圆、抛物、具耗散效应的流体方程提供研究框架-变分原理或正则逼近研究弱解的存在性，通过 De Giorgi 迭代、Nash-Moser 迭代等经典数学方法研究弱解正则性。本次报告拟从函数论观点出发,不同类型的 PDE 决定了函数 Fourier 谱集演化对应的不同规律与作用,从而获得 PDE 解的动力学行为与奇性传播规律。 另一方面, 从调和分析与 PDEs 领域的公开问题出发，分析这些问题核心困难之症结，为年轻数学工作者提供一些可能研究途径与思考。