

STABLE ALGORITHMS CANNOT RELIABLY FIND ISOLATED PERCEPTRON SOLUTIONS

Speaker: Shuyang Gong
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Time&Venue:

SCMS Room 102, May 15 (Friday), 10:30-11:30

Abstract:

We study the binary perceptron, a random constraint satisfaction problem that asks to find a Boolean vector in the intersection of independently chosen random halfspaces. A striking feature of this model is that at every positive constraint density, it is expected that a $1 - o_N(1)$ fraction of solutions are strongly isolated, i.e., separated from all others by Hamming distance $\Omega(N)$. At the same time, efficient algorithms are known to find solutions at certain positive constraint densities. This raises a natural question: can any isolated solution be algorithmically visible?

We answer this in the negative: no algorithm whose output is stable under a tiny Gaussian resampling of the disorder can reliably locate isolated solutions. We show that any stable algorithm has success probability at most ≈ 0.84233 . Furthermore, every stable algorithm that finds a solution with probability $1 - o_N(1)$ finds an isolated solution with probability $o_N(1)$. The class of stable algorithms we consider includes degree- D polynomials up to $D \leq N^{1-o(1)}$; under the low-degree heuristic (Hopkins, 2018), this suggests that locating strongly isolated solutions requires running time $\exp(\tilde{N})$. Our proof does not use overlap gap property. Instead, we show via Pitt's correlation inequality that after a random perturbation of the disorder, the number of solutions located close to a pre-existing isolated solution cannot concentrate at 1. This talk is based on joint work with Brice Huang (Stanford), Shuangping Li (Yale) and Mark Sellke (Harvard).

Biography:

Shuyang Gong is a fifth-year PhD student at Peking University, advised by Professor Dayue Chen and Professor Jian Ding. His research focuses on probability theory, particularly high-dimensional phenomena, random graph theory, and their intersections with high-dimensional statistics.