上海数学中心
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MATHEMATICAL SCIENCES

Tight Upper Bound on the Clique Size in the Square of 2-degenerate Graphs

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Abstract:

The *square* of a graph G, denoted G^2, has the same vertex set as G and has an edge between two vertices if the distance between them in G is at most 2. In general, $\Delta(G) + 1 \leq \chi(G^2) \leq \Delta(G)^2 + 1$ for every graph G. Charpentier(2014) asked whether $\chi(G^2) \leq 2\Delta(G)$ if $mad(G) < 4$. But Hocquard, Kim, and Pierron (2019) answered his question negatively. For every even value of $\Delta(G)$, they constructed a 2-degenerate graph G such that $\omega(G^2) = \frac{5}{2}\Delta(G)$. Note that if G is a 2-degenerate graph, then $mad(G) < 4$. Thus, we have that

$$\frac{5}{2}\Delta(G) \leq \max\{\chi(G^2) : G \text{ is a 2-degenerate graph}\} \leq 3\Delta(G) + 1.$$

So, it was naturally asked whether there exists a constant D_0 such that $\chi(G^2) \leq \frac{5}{2}\Delta(G)$ if G is a 2-degenerate graph with $\Delta(G) \geq D_0$. Recently Cranston and Yu (2024) showed that $\omega(G^2) \leq \frac{5}{2}\Delta(G) + 72$ if G is a 2-degenerate graph, and $\omega(G^2) \leq \frac{5}{2}\Delta(G) + 60$ if G is a 2-degenerate graph with $\Delta(G) \geq 1729$. We show that there exists a constant D_0 such that $\omega(G^2) \leq \frac{5}{2}\Delta(G)$ if G is a 2-degenerate graph with $\Delta(G) \geq D_0$. This upper bound on $\omega(G^2)$ is tight. This is joint work with Xiaopan Lian (Nankai University).