


复旦大学数学科学学院
数学综合报告会

报告题目：From Gaussian to Gumbel: extreme eigenvalues of complex Ginibre products with exact rates

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时间：2026-05-16 星期六 9:30--10:30

地点：光华楼东主楼 2001

报告摘要：

We consider the product of \(k_{n}\) independent \(n\times n\) complex Ginibre matrices and denote its eigenvalues by \(Z_{1},\ldots ,Z_{n}\). Let \(\alpha = \lim_{n\to\infty} n / k_{n}\). Using the determinantal point process method, we reduce the study of extremal eigenvalues to the evaluation of determinants of certain \(n \times n\) matrices. In the modulus case, rotational invariance makes the relevant matrix diagonal, which yields a product representation in terms of Gamma tail probabilities. In the real-part case, the matrix is no longer diagonal; we handle this by a polar-coordinate reduction that introduces an independent uniform angle and leads to explicit formulas involving Gamma variables and trigonometric integrals. After appropriate rescaling, the spectral radius \(\max_{1\leq j\leq n}|Z_{j}|\) converges weakly to a nontrivial distribution \(\Phi_{\alpha}\) when \(\alpha \in (0, +\infty)\), to the Gumbel distribution when \(\alpha = +\infty\), and to the standard normal distribution when \(\alpha = 0\). The family \(\{\Phi_{\alpha}\}_{\alpha >0}\) extends continuously to the boundary regimes: \(\Phi_{\alpha}\) converges weakly to the standard normal law as \(\alpha \to 0^{+}\) and to the Gumbel law as \(\alpha \to +\infty\). Thus the three limiting regimes are connected by the single parameter \(\alpha\), yielding a continuous transition from Gaussian to Gumbel distribution. For the spectral radius, we obtain the exact rates of convergence both in the fixed-\(\alpha\) regime and at the boundaries \(\alpha = 0\) and \(\alpha = +\infty\). For the rightmost eigenvalue \(\max_{1\leq j\leq n}\Re Z_{j}\), we establish the convergence rates in the boundary regimes, while for \(\alpha \in (0, +\infty)\) we show that the limiting distribution, though not available in closed form, still interpolates continuously between the normal and Gumbel laws.