复旦大学数学科学学院
数学综合报告会

报告题目： \ell^0-constrained Tikhonov regularization

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时间：2026-11-04 星期三 13:30-14:30

地点：光华东主楼1601

报告摘要：

We consider linear inverse problems given by an operator equation $Tu = g$ with $T : \ell^2 \to \mathcal{Y}$, where $\mathcal{Y}$ is a Hilbert space and the ground truth $u^\dagger \in \ell^2$ is assumed or known to be sparse, i.e., $\|u^\dagger\|_{\ell^0} < \infty$. We aim to reconstruct $u^\dagger$ from noisy data $g^\delta \in \mathcal{Y}$ by the ℓ^0-constrained optimization problem

$$\min_{u \in \ell^2} \|Tu - g^\delta\|_{\mathcal{Y}}^2 + \alpha \|u\|_{\ell^2}^2 \quad \text{s.t.} \quad \|u\|_{\ell^0} \le K, \tag{1}$$

where $K \in \mathbb{N}$ is the constraint parameter and $\alpha > 0$ is a small regularization parameter.

In order to solve (1) we use the equivalent reformulation

$$\|u\|_{\ell^0} \le K \quad \Leftrightarrow \quad \|u\|_{\ell^1} - |u|_K = 0$$

from [1, 2], with the so-called largest-K-norm $|u|_K := \sup_{I \subset \mathbb{N}: \#I \le K} \sum_{i \in I} |u_i|$. The associated penalized problem

$$\min_{u \in \ell^2} \|Tu - g^\delta\|_{\mathcal{Y}}^2 + \alpha \|u\|_{\ell^2}^2 + \rho \left(\|u\|_{\ell^1} - |u|_K \right), \tag{2}$$

where $\rho > 0$ is a penalty parameter, is now of difference-of-convex (DC) form. These kind of problems can be approached by the well-known DC-algorithm.

In this talk we present theoretical results on the connection between solutions and stationary points of (2) and (1), including an exact penalty result. Moreover, we show theoretical guarantees regarding the quality of computed solutions of our problem-tailored DC-algorithm with respect to the original problem (1).

We test our algorithm in a simulation study on sparse images with additional Gaussian and Poisson noise and compare the performance to a ℓ^1-penalized approach, which is often used for sparse linear inverse problems. As a real world application we apply this approach to super-resolution microscopy to locate DNA origami [3, Section 1.2].