A UNIFORMLY ACCURATE DIFFEOMORPHIC EMBEDDING METHOD FOR SOLVING EIKONAL EQUATIONS ON IMPLICIT SURFACES

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Abstract: We propose an embedding method to solve the eikonal equation on implicit surfaces, where implicit surfaces are defined by signed distance functions. Building upon a recently established diffeomorphic embedding method for hyperbolic conservation laws, we introduce a novel anisotropic eikonal equation within a tubular neighborhood of an implicit surface to approximate the eikonal equation defined on the implicit surface. Our primary contribution lies in a formulation to ensure that the solution of the Hamilton-Jacobi equation of anisotropic eikonal type remains constant along the normal direction of the surface. To solve the resulting equation numerically, we employ the Lax-Friedrichs based high-order weighted essentially non-oscillatory fast sweeping method, and we further introduce two effective singularity factorization approaches to address the upwind singularity near the point source. While standard singularity factorization formulations apply only to point-source singularities, our new methods accommodate line-source singularities when the boundary condition is extended along the normal direction of the surface. These new factorization strategies enable us to compute uniformly high-order accurate solutions for the eikonal equation on an implicit surface. Furthermore, our method can be readily extended to handle anisotropic eikonal equations or general static Hamilton-Jacobi equations defined on implicit surfaces. Two- and three-dimensional examples demonstrate the accuracy and efficiency of our new embedding method.