L^q-SPECTRA OF DYNAMICALLY DRIVEN SELF-SIMILAR MEASURES: THE MULTI-DIMENSIONAL CASE

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Time：Mon, Mar 10th, 15:00-16:00

Venue：Room 102, SCMS

Abstract： A great deal of interest in fractal geometry centres on determining the dimensional properties of self-similar sets and measures, as well as of their projections and convolutions. In a seminal contribution dating from nearly a decade ago, Hochman achieved substantial progress towards the celebrated exact overlaps conjecture, establishing that the Hausdorff dimension of self-similar sets and measures on the real line matches the similarity dimension whenever the generating iterated function system satisfies exponential separation. The result was subsequently refined by Shmerkin, who established the analogue for the full L^q-spectrum of self-similar measures and successfully applied it to settle long-standing conjectures in dynamics and fractal geometry, most notably Furstenberg's intersection conjecture for the action of multiplicatively independent integers on the torus. In joint work with Shmerkin, we extend the dimensional result to any ambient dimension under an additional unsaturation assumption; as in the one-dimensional case, our framework consists of the class of dynamically driven self-similar measures, which allows for a unified treatment of self-similar and stochastically self-similar measures, their projections and convolutions. The argument relies crucially on an inverse theorem for the L^q-norm of convolutions of discrete measures in Euclidean spaces, recently established by Shmerkin, akin in spirit to the asymmetric version of the Balog-Szemerédi-Gowers theorem due to Tao and Vu.