复旦大学数学科学学院

数学综合报告会

午间学术报告会（一百九十四）

报告题目：Axioms, definable sets, and dividing lines: a tour of model theory

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报告地点：光华东主楼 2201

摘要:

在Model theory is a subjectwhich lies in the intersection of algebra and logic.Given a structure M, one can lookfor a set of axioms which completely pin down the (first-order)properties ofM.For example,the field (C,+,) is completely axiomatized by the axioms of algebraically closed fields of characteristic 0.To axiomatize a structure M, it is usually necessary to understand the “ definable sets ” ofM,that is,the subsets DMn which are defined by (first-order)formulas.For example, in the field C,the definable sets are the constructible set s of algebraic geometry.The real exponential field (R,+,,exp) has much more complicated definable sets. Nevertheless, it has a special property called “ o-minimality ” which implies thatthe definable sets have finite t riangulations, and definable functions are piecewise smooth, amongmany other things.

In the process of axiomatizing structures and analyzing definable sets,model theorists have uncovered a number of “ dividing lines ” such as stability,NIP, simplicity, etc.These dividing lines reflect natural dichotomies in model theory.For example, if a theory T is unstable,then T has the maximumpossible number of models, and ifT is stable,then the models ofT have a natural notion of “ independence ” similar to algebraic independence in C.The class ofNIPtheories generalizes the stable and o-minimal theories, includes many natural mathematical structures like R and Qp, and is closely connected to the concept ofVC-dimension i n machine learning.

My own research touches on each ofthese areas. I will discuss my partial results towards the classification of NIPfields and rings, as well as my workwith Tran, Walsberg, and Ye on the model theory of large fields.

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