THE LOGARITHMIC MINKOWSKI PROBLEM IN $\mathbb{R}^2$

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Time：Tue, Apr. 30th, 14:30-15:00
Venue: Room 106, SCMS

Abstract: The classical Minkowski's existence theorem due to Minkowski, Aleksandrov and Fenchel-Jessen characterizes the surface area measure S_K of a convex body K in $\mathbb{R}^{\boldsymbol{n}}$. More precisely, it solves the Monge-Ampère equation

$$\det(\nabla^2 h + h\mathrm{Id}) = f$$

on the unit sphere $\mathbb{S}^{n-1}$, where a convex body K with C_+^2 boundary provides a solution if $h = h_K$ for the support function h_K of K.

The logarithmic Minkowski problem

$$h\det(\nabla^2 h + h\mathrm{Id}) = f$$

was posed by Firey in his 1974 seminal paper. It seeks to characterize the cone volume measure $dV_K = \frac{1}{n} h_K dS_K$ of a convex body K containing the origin. The logarithmic Minkowski problem is a challenging problem in convex geometry and receives much attention since 2012.

In this talk, we will present our very recent work on the logarithmic Minkowski problem. A necessary condition for the existence of solutions to the logarithmic Minkowski problem in $\mathbb{R}^{\boldsymbol{2}}$, which turns to be stronger than the celebrated subspace concentration condition, is given. The sufficient and necessary conditions for the existence of solutions to the logarithmic problem for quadrilaterals, as well as the number of solutions, are fully characterized.

This talk is based on the joint work with Yude Liu, Qiang Sun and Ge Xiong.